**Thomas-Fermi model Electric Susceptibility**

I’m basically just reproducing a file in Stat Mech folder. We’ll derive the static TF result using the Boltzman equation (in RTA approximation), and then go on to getting dynamic susceptibility. So I’d like to get a non-perturbative equation for the current, starting from the RTA equation (we could do this perturbatively like in other cases, but opting for this approach instead for variation’s sake):



Well no need to do this all over again. Just see Stat Mech folder/RTA equation (MF Solution). And we find two self-consistent equation,



where ρ is charge density, and current density is **j** = ρ**u** (where **u** is the local velocity of the particles) Now would like to use it to take a look at the electric susceptibility. The susceptibility, χirr, is defined as the charge density response to an electric field perturbation.



We can work this out from our equation. Let j = 0, and B = 0 too. We’ll assume we have an isotropic medium, so **D** = D**1**, and we’ll call ρind = en. Then we have:



Now we can’t exactly lift the gradient from both sides because n in the numerator is presumably position dependent, as ρ is clearly supposed to have a non-zero gradient. But if we restrict our aspirations to just getting an equation for ρind correct to first order in φ, then we can treat that n as a global constant. And if so, then we can say,



So we have:



Now we’ll fill in D, the 3d, T = 0 version,



Now use the 3D result (see Condensed Matter/Free Day/Electrons/Properties),



And we can say,



So we arrive at the familiar Thomas-Fermi susceptibility.



We can define a generalized susceptibility. Say we have a potential perturbation φ(r,t) = Re[φ(q,ω)eiq·r-iωt], and an induced charge density response ρind(r,t) = Re[ρind(q,ω)eiq·r-iωt], then electric susceptibility is defined via the proportionality:



We can work this out, but we’ll need the time-dependent RTA equation – still no B field. And we’ll take the low scattering limit, i.e., τsc → ∞. In this case, the non-zero/dominant terms are:



Well we need to relate jind and ρind. We have the continuity equation,



So let’s take the divergence of both sides of our RTA equation,



Then we can say,



Now fill in our expressions for ρind and φ,



So we have:



Filling in D = 2τscεF/3m again, we’ll get:



So we have:

